Surrogate ratio methodology for the indirect determination of neutron capture cross sections

B. L. Goldblum* and S. G. Prussin
Department of Nuclear Engineering, University of California, Berkeley, California 94720, USA

L. A. Bernstein and W. Younes
Lawrence Livermore National Laboratory, Livermore, California 94551, USA

M. Guttormsen and H. T. Nyhus
Department of Physics, University of Oslo, Post Office Box 1048 Blindern, N-0316 Oslo, Norway

(Received 2 December 2009; published 18 May 2010)

The relative $\gamma$-decay probabilities of the $^{162}$Dy to $^{164}$Dy and $^{162}$Dy to $^{164}$Dy residual nuclei, produced using light-ion-induced direct reactions, were measured as a function of excitation energy using the CACTUS array at the Oslo Cyclotron Laboratory. The external surrogate ratio method (SRM) was used to convert these relative $\gamma$-decay probabilities into the $^{164}$Dy($n,\gamma$) cross section in an equivalent neutron energy range of 130–560 keV. The directly measured $^{160}$Dy($n,\gamma$) cross section, obtained from the Evaluated Nuclear Data Files (ENDF/B-VII.0), was compared to the experimentally determined surrogate $^{164}$Dy($n,\gamma$) cross section obtained using compound-nucleus pairs with both similar ($^{162}$Dy to $^{164}$Dy) and dissimilar ($^{162}$Dy to $^{161}$Dy) nuclear structures. A $\gamma$-ray energy threshold was identified, based upon pairing gap parameters, that provides a first-order correction to the statistical $\gamma$-ray tagging approach and improves the agreement between the surrogate cross-section data and the evaluated result.

DOI: 10.1103/PhysRevC.81.054606  PACS number(s): 24.10.–i, 24.87.+y, 25.55.–e, 25.40.Lw

I. INTRODUCTION

The surrogate ratio method (SRM) is a technique for the indirect determination of neutron-induced reaction cross sections on both stable and radioactive nuclei [1,2]. The surrogate ratio approach involves the use of a light-ion induced direct (or “surrogate”) reaction to form the same compound nucleus produced in the neutron-induced reaction of interest. Relative compound nuclear decay probabilities are determined, and the unknown neutron-induced reaction cross section is then deduced relative to one that is well measured. The SRM is a promising technique, in that neutron-induced fission cross sections have been obtained using the SRM from energies of several hundred keV up to 20 MeV with total uncertainties on the order of 10% [2–4].

Neutron capture cross sections on radioactive nuclei, many of which are difficult to obtain directly, are important input data for a number of applications, including advanced nuclear reactor performance calculations [5] and the study of astrophysical phenomena [6]. Recent work has been undertaken to explore the applicability of the SRM in the determination of neutron capture cross sections [7–9]. There are two means of application of the SRM: internal and external. In the internal SRM, relative decay probabilities of the same compound nucleus are measured (e.g., fission relative to $\gamma$ decay) and the unknown cross section is extracted relative to one that is well known. Recent experimental data showcase the first application of the internal SRM and suggest surrogate ($n,\gamma$) cross-section data may be obtained to within approximately 20% of directly measured results down to 1 MeV equivalent neutron energy [8].

The external SRM involves the use of the same surrogate direct reaction on two different target nuclei. The compound nuclear decay probabilities for two different compound nuclei into the same exit channel (e.g., $\gamma$ decay) are determined relative to one another. In a previous article [9], the external SRM was first employed in the determination of the $^{170}$Yb($n,\gamma$) cross section relative to the $^{160}$Dy($n,\gamma$) cross section, using both ($^3$He,$^3$He) and ($^3$He,$^\alpha$) surrogate reactions. This approach involved compound nuclei pairs with similar nuclear structure (i.e., both even-odd compound nuclei) and similar target pairs (i.e., both even-even or both even-odd nuclei, with similar deformation and mass). The surrogate $^{170}$Yb($n,\gamma$) cross sections obtained via both the ($^3$He,$^3$He) and ($^3$He,$^\alpha$) surrogate reactions exhibited remarkable agreement with the directly measured result.

Assuming that the two nuclei used in forming the ratio are sufficiently similar, the SRM has the advantage that it minimizes correlated errors in the interpretation of the experimental data [1,2]. Correlated errors could arise from pre-equilibrium decay, fission-fragment anisotropies, and effects of the angular-momentum population distribution, among other factors. However, deviations of the surrogate cross section compared to directly measured data have been observed when two compound nuclei with dissimilar nuclear structure are compared in the SRM [10].

To explore the viability of the SRM for deducing neutron capture cross sections, we compare the directly measured $^{161}$Dy($n,\gamma$) cross section, obtained from the Evaluated Nuclear Data Files (ENDF/B-VII.0) [11], to the experimentally determined surrogate $^{161}$Dy($n,\gamma$) cross section obtained using both

*bethany@nuc.berkeley.edu
similar and dissimilar compound nuclei pairs. In Sec. IV A, the surrogate $^{161}\text{Dy}(n,\gamma)$ cross section is obtained using similar compound nuclei pairs, $^{162}\text{Dy}$ and $^{164}\text{Dy}$, accessed via the $(^3\text{He},^3\text{He}')$ inelastic scattering reaction. In Sec. IV B, we consider dissimilar compound nuclei pairs ($^{162}\text{Dy}$ and $^{161}\text{Dy}$) accessed via both the $(^3\text{He},^3\text{He}')$ and $(^3\text{He},\alpha)$ reactions. In this latter case, surrogate measurements were performed using both the $(^3\text{He},^3\text{He}')$ and $(^3\text{He},\alpha)$ direct reactions to explore the effect of entrance channel on the extracted surrogate ratio. The surrogate reactions used to access the compound nuclei of interest and the corresponding neutron-induced entrance channels are summarized in Table I.

### II. THEORY

In a recent report [9], we derived an expression for an unknown neutron capture cross section of interest, $\sigma^{(1)}_{(n,\gamma)}$, obtained as a function of excitation energy, $E$, in the Weisskopf-Ewing limit relative to some well-measured cross section, $\sigma^{(2)}_{(n,\gamma)}$,

$$
\sigma^{(1)}_{(n,\gamma)}(E) = A \frac{N^{(1)}_{\beta\gamma}(E)}{N^{(2)}_{\beta\gamma}(E)} \sigma^{(2)}_{(n,\gamma)}(E),
$$

where

$$
A = \frac{N^{(2)}_{\beta\gamma}(E_A)}{N^{(1)}_{\beta\gamma}(E_A)}.
$$

The superscripts (1) and (2) denote the two different compound nuclei employed in the surrogate ratio analysis. $N_{\beta\gamma}(E)$ represents the number of $\gamma$-decay events in coincidence with the surrogate reaction ejectile as a function of excitation energy in the compound nucleus. The constant $A$ is a normalization parameter, where $N_{\beta\gamma}(E_A)$ represents the number of particle-$\gamma$ coincident events at some excitation energy, $E_A$, just below the neutron separation energy, where the only open decay channels are electromagnetic transitions and associated processes (e.g., internal conversion). That is, $E_A = S_n - \delta$, where $S_n$ is the neutron separation energy in the compound nucleus and $\delta$ is the amount of energy needed to ensure that the total excitation energy, $E_A$, is sufficiently below the neutron separation energy within experimental uncertainty such that essentially the only open decay channel is $\gamma$-ray emission. Thus, the $A$ parameter accounts for experimental conditions such as the integrated beam current and number of target atoms for the two reactions of interest. In the final surrogate cross-section data presented in Secs. IV A and IV B, the $\sigma^{(1)}_{(n,\gamma)}(E)$ data are translated into equivalent neutron energy, $E_n$, defined as the energy of the neutron in the desired reaction and related to the excitation energy of the compound nucleus, $E$, by $E_n = E - S_n^{(1)}$, where $S_n^{(1)}$ is the separation energy of the neutron in the compound system.

Equation (1) is predicated on the following assumptions:

(i) The compound nuclear decay probabilities are independent of the total angular momentum, $J$, and parity, $\pi$, of the populated states. This is the Weisskopf-Ewing limit [12] of the Hauser-Feshbach theory, a fundamental assumption of the SRM. The Weisskopf-Ewing approximation requires that the energy of the compound nucleus is sufficiently high that nearly all channels into which it can decay are dominated by integrals over the level density (i.e., the fraction of decays proceeding to resolved states is small).

(ii) The neutron-induced formation cross sections for the two compound nuclei formed in the ratio are sufficiently similar over the excitation energy range considered for the measurement such that they cancel in the ratio with negligible uncertainty. This is reasonable given that the optical model potential parameters vary slowly for Dy nuclei [13].

(iii) The $(n,n'\gamma)$ channel, where photons are emitted after neutron evaporation, is insignificant in the excitation energy range relevant for the surrogate measurement or sufficiently similar for the two nuclei employed in the ratio such that contributions from this decay channel cancel in the ratio. A $\gamma$-ray energy threshold of 500 keV can be applied, for example, to ensure that contributions from the $(n,n'\gamma)$ channel are excluded in the particle-$\gamma$ coincidence spectra for excitation energies up to 500 keV above the neutron binding energy. The effect of application of the $\gamma$-ray energy threshold on the extracted surrogate cross section is further discussed in Sec. V.

(iv) The cross sections for the direct reactions to form the compound nuclei via the two surrogate reactions employed in the ratio are equal at and near the neutron separation energy. This is a reasonable assumption when target pairs with similar nuclear structure are considered (both even-even nuclei or both even-odd nuclei, with similar deformation and mass), but may fail in the consideration of dissimilar target pairs.

---

**TABLE I. Summary of relevant surrogate reactions, compound nuclei, and corresponding neutron-induced entrance channels.**

<table>
<thead>
<tr>
<th>Surrogate entrance channel</th>
<th>Compound nucleus</th>
<th>Neutron-induced entrance channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{161}\text{Dy}(^3\text{He},^3\text{He}')$</td>
<td>$^{161}\text{Dy}$</td>
<td>$^{160}\text{Dy} + n$</td>
</tr>
<tr>
<td>$^{162}\text{Dy}(^3\text{He},^3\text{He}')$</td>
<td>$^{162}\text{Dy}$</td>
<td>$^{161}\text{Dy} + n$</td>
</tr>
<tr>
<td>$^{164}\text{Dy}(^3\text{He},^3\text{He}')$</td>
<td>$^{164}\text{Dy}$</td>
<td>$^{162}\text{Dy} + n$</td>
</tr>
<tr>
<td>$^{162}\text{Dy}(^3\text{He},\alpha)$</td>
<td>$^{162}\text{Dy}$</td>
<td>$^{162}\text{Dy} + n$</td>
</tr>
<tr>
<td>$^{163}\text{Dy}(^3\text{He},\alpha)$</td>
<td>$^{162}\text{Dy}$</td>
<td>$^{162}\text{Dy} + n$</td>
</tr>
</tbody>
</table>
These assumptions were successfully employed in a previous study [9] of the surrogate $^{170}$Yb($n,\gamma$) cross section in an equivalent neutron energy range of 165 to 465 keV, where the compound nuclei, $^{171}$Yb and $^{161}$Dy, had similar nuclear structure (both even-odd nuclei) and similar deformation and mass. Although a breakdown of the Weisskopf-Ewing approximation is expected to be significant for $E_n \approx 0$–3 MeV [14] and has been experimentally observed in the neutron energy range of 0.6 to 1.9 MeV [10], the SRM is expected to lessen the effect of correlated errors for two similar compound nuclei. That is, if a deviation from the Weisskopf-Ewing approximation is similar for the two nuclei employed in the ratio, a partial cancellation of these effects is expected. In this analysis, we employ similar (both even-even compound nuclei) and dissimilar compound nuclei pairs (an even-even nucleus and an even-odd nucleus). For compound nuclei pairs with dissimilar nuclear structure, cancellation of correlated effects is likely diminished and any residual dependence of the decay probabilities on the angular momentum population distribution is more likely to be manifest in the extracted surrogate cross section.

III. EXPERIMENTAL APPARATUS

The experiments on targets of $^{161,162,163}$Dy were carried out with 45-MeV $^3$He ions at the Oslo Cyclotron Laboratory and have been reported earlier [15,16]. The duration of these experiments were 1–2 weeks with beam currents of 2 nA. The $^{161,162,163}$Dy targets were self-supporting and isotopically enriched to approximately 95% in the isotope of interest with thicknesses of approximately 2 mg/cm$^2$. An 11-day experiment on the $^{164}$Dy target was carried out with 38-MeV $^3$He ions at the Oslo Cyclotron Laboratory. The $^{164}$Dy target, enriched to approximately 98.5% in the isotope of interest, was prepared as a self-supporting foil of approximately 1.7 mg/cm$^2$ thickness.

Particle-$\gamma$ coincident events for the relevant compound nuclei were measured with the CACTUS multidetector array [17]. Charged particle ejectiles were detected with eight particle telescopes, each consisting of a front Si $\Delta E$ detector and a back Si(Li) $E$ detector with thicknesses of 140 and 3000 $\mu$m, respectively, placed at an angle of 45$^\circ$ relative to the beam direction. The intrinsic particle detector energy resolution was determined using an $^{241}$Am source to be 50 keV at $E_a = 5.486$ MeV. The average energy resolution of the particle detectors, dominated by the systematic uncertainty arising from determination of the recoil angle of the residual nucleus, was determined to be 200 keV over the entire particle energy region. An array of 28 NaI $\gamma$-ray detectors, which intercepted a solid angle of 15% of 4$\pi$, surrounded the target and particle detectors. The full width at half maximum of the detector output varied from 80 keV at a $\gamma$-ray energy of 1.33 MeV up to 250 keV at a $\gamma$-ray energy of 8 MeV. The response function for the CACTUS array as a function of $\gamma$-ray energy was previously determined [18] and accounted for in the extracted particle-$\gamma$ coincidence spectra [19].

IV. DATA ANALYSIS AND RESULTS

A two-dimensional, particle-gated matrix of $\gamma$-ray energy and compound nuclear excitation energy, $E$ (as determined by the ejectile energy), was recorded for each reaction of interest. For each initial excitation energy, $\gamma$-ray spectra covering an energy range of 100 keV to approximately 6 MeV were recorded. The deexcitation of a given level populated in the compound nucleus may result in a cascade of $\gamma$-ray emission, involving a certain number of levels (each with an associated $\gamma$ ray). To identify the $\gamma$-decay channel, only the first $\gamma$ ray emitted is of interest as the secondary $\gamma$ rays increase the ostensible number of $\gamma$-decay events. To account for this phenomenon, the $\gamma$-ray multiplicity, $M_{\gamma}(E)$, was determined as a function of excitation energy, $E$, for each compound nucleus for excitation energies up to the neutron separation energy using the method described in Ref. [20]. A quadratic fit of the deduced $M_{\gamma}(E)$ between $E \approx 2$ MeV and $E = S_n$ was performed and extrapolated for $E > S_n$. The extrapolated $\gamma$-ray multiplicities determined in this manner were found to be consistent with those calculated using the model of Dissing and Vigezzi [21].

Each $\gamma$-ray spectrum was summed over a range of $\gamma$-ray energies for each 120-keV bin of compound nuclear excitation energy to form the particle-$\gamma$ coincidence spectra. For the data employed in the surrogate cross-section analysis, a lower $\gamma$-ray energy threshold, $E_{\text{th}}$, was applied to ensure that contributions from the $(n,n^\prime\gamma)$ channel are omitted in the particle-$\gamma$ coincidence spectra for the excitation energies relevant for the surrogate measurement ($E$ between $S_n$ and $S_n + E\text{th}$). Application of this $\gamma$-ray energy threshold also excludes some $\gamma$ rays from the analysis that would have been appropriately attributed to the $(n,\gamma)$ channel. This is further discussed in Sec. V.

A. Evaluation of the surrogate $^{164}$Dy($n,\gamma$) cross section using similar compound nuclei pairs

As outlined in Eq. (1), the ratio of the number of particle-$\gamma$ coincident events to produce two different compound nuclei (in this case, $^{164}$Dy and $^{166}$Dy) using the same surrogate reaction, $(^3\text{He},^3\text{He})$, was determined. The ratio was normalized, as shown in Eq. (2), at $E_A = 7100$ keV. To obtain the $^{161}$Dy($n,\gamma$) cross section, the normalized ratio data were multiplied by the ENDF/B-VII.0 $^{163}$Dy($n,\gamma$) cross section matched at excitation energy as described in Eq. (1). The result was then shifted into equivalent neutron energy by subtracting the neutron separation energy in the $^{162}$Dy compound nucleus ($S_n = 8196.9$ keV) from the excitation energy. The surrogate $^{161}$Dy($n,\gamma$) cross section with a $\gamma$-ray energy threshold of 500 keV and 2 MeV is shown in Figs. 1(a) and 1(b), respectively. The solid line represents the evaluated $^{164}$Dy($n,\gamma$) cross section obtained from ENDF/B-VII.0.

The 500-keV $\gamma$-ray energy threshold applied to the data in Fig. 1(a) ensures that contributions from the $(n,n^\prime\gamma)$ channel are excluded in the $^{164}$Dy($n,\gamma$) surrogate cross-section data. Yet, the surrogate data remain systematically lower than the evaluated result within the total estimated uncertainty, with the exception of the data point at 250 keV. With application of the
FIG. 1. The $^{161}$Dy($n,\gamma$) cross section extracted using the SRM with similar compound nuclei pairs relative to the evaluated $^{163}$Dy($n,\gamma$) cross section obtained from ENDF/B-VII.0 as a function of equivalent neutron energy obtained via the ($^3$He,$^3$He$'$) inelastic scattering reaction with a $\gamma$-ray energy threshold of (a) 500 keV and (b) 2 MeV. The error bars represent both the statistical and nonstatistical uncertainty. For comparison, the directly measured $^{161}$Dy($n,\gamma$) cross section from ENDF/B-VII.0 is denoted by the solid line.

2-MeV $\gamma$-ray energy threshold, the surrogate data converge with the evaluated result within total estimated uncertainty over the entire equivalent neutron energy range probed. The physical interpretation is discussed in Sec. V.

B. Evaluation of the surrogate $^{161}$Dy($n,\gamma$) cross section using dissimilar compound nuclei pairs

Similar to the analysis in Sec. IV A, the ratio of the number of particle-$\gamma$ coincident events to produce two different compound nuclei (in this case, $^{162}$Dy and $^{161}$Dy), using the same surrogate reaction, either ($^3$He,$^3$He$'$) or ($^3$He,$\alpha$), was determined. The ratio was normalized, as shown in Eq. (2), at $E_A = 6120$ keV and $E_A' = 6180$ keV, for the ($^3$He,$^3$He$'$) and ($^3$He,$\alpha$) surrogate ratio data, respectively. To obtain the $^{161}$Dy($n,\gamma$) cross section, the normalized ratio data were multiplied by the ENDF/B-VII.0 $^{160}$Dy($n,\gamma$) cross-section data matched at excitation energy as described in Eq. (1). The result was then shifted into equivalent neutron energy by subtracting the neutron separation energy in the $^{162}$Dy compound nucleus from the excitation energy. The surrogate $^{161}$Dy($n,\gamma$) cross-section data obtained using the ($^3$He,$^3$He$'$) surrogate reaction (open circles) and the ($^3$He,$\alpha$) surrogate reaction (solid squares) with a $\gamma$-ray energy threshold of 500 keV and 2 MeV are shown in Figs. 2(a) and 2(b), respectively. The solid line represents the directly measured $^{161}$Dy($n,\gamma$) cross section obtained from ENDF/B-VII.0. Though it may seem counterintuitive that the error bars on the surrogate cross-section data decrease with increasing equivalent neutron energy, the uncertainty on the data is largely due to nonstatistical effects arising from subtraction of potential contaminant contributions from neighboring isotopes. The sources of uncertainty in the surrogate cross-section data are outlined in detail in the Appendix.

The SRM $^{161}$Dy($n,\gamma$) cross section obtained using dissimilar compound nuclei pairs was found to be independent of the type of surrogate reaction employed, within the total uncertainty, indicating no significant entrance channel effects. Although application of the 500-keV $\gamma$-ray energy threshold excludes contributions from the ($n,n'\gamma$) channel up to an equivalent neutron energy of 500 keV, again the surrogate data in Fig. 2(a) are systematically lower than the evaluated result. With a 2-MeV $\gamma$-ray energy threshold, the $^{161}$Dy($n,\gamma$) surrogate cross section obtained using dissimilar compound nuclei pairs agreed with the evaluation within the total estimated uncertainty over the entire equivalent neutron energy range probed, with the exception of a slight deviation of the 500-keV ($^3$He,$\alpha$) surrogate data point as compared to the evaluation. In the Appendix, we detail the uncertainties shown in Figs. 1 and 2. The effect of the $\gamma$-ray energy threshold on the surrogate cross-section data is further discussed later in this article.

V. DISCUSSION

For even-even nuclei, as a result of the pairing effect, an energy gap exists in the nuclear energy levels where below this gap relatively few collective states exist. For even-odd nuclei,
measured the wave functions describing the states below the energy gap are quasi-single-particle, or in the case of deformed nuclei, admixtures of relatively few shell model basis states (Nilsson model). Above the energy gap, a high level density region exists. As outlined in Sec. II Assumption (i), the Weisskopf-Ewing limit of the Hauser-Feshbach theory is a statistical approximation, not applicable when the individual final states are well resolved, such as below the energy gap, in the few MeV of excitation energy where the level density is low.

Positive equivalent neutron energy in the surrogate analysis occurs at excitation energies greater than the neutron separation energy. To determine the number of $\gamma$-decay events in coincidence with the surrogate reaction ejectile as a function of excitation energy in the compound nucleus $[N_{\gamma}(E) \text{ in Eq. (1)}]$, the total number of detected $\gamma$ rays resulting from the cascade of $\gamma$-ray emission following deexcitation of a nuclear level at positive equivalent neutron energy is divided by the extrapolated total $\gamma$-ray multiplicity, $M_{\gamma}(E)$. The $\gamma$-ray energy threshold ensures that $\gamma$-ray transitions with an energy less than $E_{\text{th}}$ are excluded from the analysis (i.e., the discrete transitions between well-resolved states at an excitation energy of $E_{\text{th}}$ and below are omitted in the analysis). However, the $\gamma$-ray energy threshold also eliminates low-energy transitions between high-lying states well above the energy gap. Further, transitions to the resolved states at excitation energies below $E_{\text{th}}$ from initial states with excitation energies above $E_{\text{th}}$ are still included in the analysis provided that $E_{\gamma} > E_{\text{th}}$. As $E_{\text{th}}$ is increased, the fraction of decays included in the analysis that proceed to resolved states decreases. The model of Dössing and Vig cheers [21] suggests that the average energy of a statistical $\gamma$ ray emitted from states at excitation energies relevant for the surrogate cross-section measurement is greater than $E_{\text{th}}$, implying that relatively few primary statistical $\gamma$-ray transitions of interest are omitted from the analysis. By applying a higher $\gamma$-ray energy threshold, $\gamma$ rays from higher excitation energies are selectively emphasized and are less likely to exhibit nonstatistical behavior.

The energy gaps, as calculated from pairing gap parameters $\Delta_p$ and $\Delta_\alpha$, evaluated from even-odd mass differences [22], for the $^{161}$Dy, $^{163}$Dy, and $^{164}$Dy compound nuclei are 0.79, 1.85, and 1.70 MeV, respectively [23]. By applying a $\gamma$-ray energy threshold greater than or equal to the energy gap for both compound nuclei employed in the surrogate ratio analysis, the fraction of decays included in the analysis that proceed to resolved states is diminished and $\gamma$ rays from the first-generation statistical cascade are emphasized. By increasing the $\gamma$-ray energy threshold from 500 keV to 2 MeV, an improved agreement between the surrogate cross-section data and the evaluation was observed, as indicated in Figs. 1 and 2. For both the similar and dissimilar compound nuclei pairs, the surrogate cross-section data converge with the evaluated result as the $\gamma$-ray energy threshold is increased from 0 to 2 MeV. Application of a $\gamma$-ray energy threshold greater than 2 MeV does not change the level of agreement between the surrogate data and the evaluation.

Reasonable agreement of the surrogate cross section obtained using similar compound nuclei pairs with the evaluated result for a $\gamma$-ray energy threshold of 500 keV as shown in Fig. 1(a) may suggest that residual dependence of the $\gamma$-decay probabilities on angular momentum is at least partially canceled in the surrogate ratio analysis. This is plausible given that the target pairs are both even-even nuclei, with similar nuclear structure, deformation, and mass. For the surrogate ratio cross-section data obtained using dissimilar compound nuclei pairs (i.e., an even-even nucleus and an even-odd nucleus), a $\gamma$-ray energy threshold is required such that the quasi-particle level density in the even-even compound nucleus will be at least as large as in the neighboring odd mass nucleus. That is, for dissimilar compound nuclei pairs, the $\gamma$-ray energy threshold must be at least as great as the energy gap in the even-even compound nucleus before reasonable agreement between the surrogate cross-section data and the evaluation is achieved.

VI. CONCLUSIONS

We have indirectly measured the $^{161}$Dy($n,\gamma$) cross section using the SRM over an equivalent neutron energy range of 130 to 560 keV using compound nuclei pairs with both similar and dissimilar nuclear structures. The formalism of the SRM, built upon the Weisskopf-Ewing approximation, requires that the fraction of decays proceeding to resolved states is small. A $\gamma$-ray energy threshold greater than or equal to the energy gap of both compound nuclei employed in the surrogate ratio analysis is required to reduce the fraction of decays proceeding to resolved states. Using a $\gamma$-ray energy threshold of 2 MeV, the $^{161}$Dy($n,\gamma$) cross section extracted using both similar and dissimilar compound nuclei pairs exhibited general agreement with the evaluated result. For the similar compound nuclei pairs, correlated nuclear structure effects resulting from a breakdown of the Weisskopf-Ewing approximation make application of the $\gamma$-ray energy threshold less critical than for the dissimilar compound nuclei pairs. With application of the 2 MeV $\gamma$-ray energy threshold, the average deviation between the surrogate and evaluated data was approximately 12% for the similar compound nuclei pairs and 15% and 23% for the dissimilar compound nuclei pairs generated via the ($^3$He,$^3$He) and ($^3$He,\alpha) surrogate reactions, respectively. To address nuclear data needs for modeling advanced nuclear energy systems and astrophysical phenomena, decreased total experimental uncertainty is desirable. Future work is needed to reduce the surrogate data uncertainty and further investigate the surrogate ratio methodology for the determination of neutron capture cross sections over a broad mass range; however, present work suggests that the technique does have substantive possibility.

ACKNOWLEDGMENTS

This work was supported, in part, by the University of California, Berkeley, Chancellor’s Postdoctoral Program and the Clare Boothe Luce Foundation and was performed under the auspices of the US Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344. The Norwegian Research Council (NFR) and Berkeley Nuclear Research Center (BNRC) are also gratefully acknowledged.
APPENDIX: UNCERTAINTY IN THE SURROGATE CROSS-SECTION MEASUREMENTS

The sources of nonstatistical uncertainty in the $^{161}\text{Dy}(n,\gamma)$ surrogate ratio cross section obtained using similar compound nuclei pairs include the uncertainty in the fiducial cross section (i.e., the $^{163}\text{Dy}(n,\gamma)$ cross section), the unfolding of the CACTUS detector response function, the energy identified as the normalization point for the $^{161}\text{Dy}(n,\gamma)$ ratio, and contributions to the particle-\(\gamma\) spectra due to target contaminants. For the $^{161}\text{Dy}(n,\gamma)$ surrogate ratio cross section obtained using dissimilar compound nuclei pairs, the measurement is obtained relative to the $^{160}\text{Dy}(n,\gamma)$ cross section. The uncertainty in the $^{160}\text{Dy}(n,\gamma)$ and $^{163}\text{Dy}(n,\gamma)$ cross sections obtained from ENDF/B-VII.0 is approximately 10–20\% in the excitation energy range relevant for the surrogate ratio cross-section measurement [24]. The 20\% value is adopted here to obtain a conservative estimate of the overall uncertainty in the surrogate cross-section data. To explore the effect of the correction for the CACTUS detector response function on the extracted data, a raw particle-\(\gamma\) coincidence spectrum was obtained and compared to the coincidence spectrum unfolded with the detector response function. The correction to the data varied by approximately 6\% as a function of excitation energy. Because correlated errors cancel in the ratio measurement, only the variation with excitation energy contributes to the systematic uncertainty in the measurement of the number of particle-\(\gamma\) coincident events.

For the similar compound nuclei pairs, the \(A\) parameter in Eq. (2) was determined to be 0.95 \pm 0.01 and 1.05 \pm 0.01 (statistical uncertainty only) at \(E_{A} = 7010\) keV for \(E_{th} = 500\) keV and 2 MeV, respectively. In the case of dissimilar compound nuclei pairs, for \(\beta = (^{3}\text{He},^{3}\text{He}')\), the \(A\) parameter in Eq. (2) was determined to be 0.85 \pm 0.01 and 0.97 \pm 0.01 at \(E_{A} = 5760\) keV for \(E_{th} = 500\) keV and 2 MeV, respectively. For \((^{3}\text{He},\alpha)\), the \(A\) parameter was determined to be 1.06 \pm 0.01 and 1.21 \pm 0.02 at \(E_{A} = 5700\) keV for \(E_{th} = 500\) keV and 2 MeV, respectively. Note that the \(A\) parameter is determined at a lower excitation energy for the dissimilar compound nuclei pairs to ensure that \(E_{A}\) is below the neutron separation energy for both nuclei employed in the ratio. A sensitivity study of the effect of \(\delta\) on the \(A\) parameter indicates that for \(\delta \leq 120\) keV, the \(A\) parameter is shifted by as much as 4\% for the similar compound nuclei pairs and 2\% and 7\% for \(\beta = (^{3}\text{He},^{3}\text{He}')\) and \((^{3}\text{He},\alpha)\), respectively, for the dissimilar compound nuclei pairs.

Isotopic target contamination presents a possible contribution to the uncertainty in the particle-\(\gamma\) coincidence spectra. For the similar compound nuclei pairs, the relevant residual nuclei resulting from reactions on contaminants are $^{163}\text{Dy}$ for the desired $^{164}\text{Dy}$ compound nucleus and $^{161}\text{Dy}$ and $^{163}\text{Dy}$ for the desired $^{162}\text{Dy}$ compound nucleus. The $^{165}\text{Dy}$ isotope is not considered as a source of contamination for the $^{164}\text{Dy}$ target because of its short half-life ($t_{1/2} = 2.334$ h). For both the $(^{3}\text{He},^{3}\text{He}')$ and $(^{3}\text{He},\alpha)$ measurements for the dissimilar compound nuclei pairs, the relevant residual nuclei resulting from reactions on contaminants are $^{164}\text{Dy}$ and $^{162}\text{Dy}$ for the desired $^{161}\text{Dy}$ compound nucleus and $^{163}\text{Dy}$ and $^{162}\text{Dy}$ for the desired $^{162}\text{Dy}$ compound nucleus. It was assumed that the shape of the particle-\(\gamma\) spectrum is the same for a pure target as for the contaminants. To quantify the effect of contaminants on the desired particle-\(\gamma\) spectra, the particle-\(\gamma\) coincidence spectrum was shifted in energy to result in a threshold at the neutron binding energy of the relevant contaminant nucleus and scaled using the target composition. Each contaminant residual nucleus was assumed to comprise 5\% of the target composition for the $^{161,162,163}\text{Dy}$ targets and 2.5\% of the $^{164}\text{Dy}$ target, representing a conservative estimate. The background-subtracted coincidence spectrum was compared to the raw coincidence spectrum and an excitation-energy-dependent percent change for the surrogate ratios was determined, with maxima of 6\% for the similar compound nuclei pairs and 78\% and 63\%, respectively, for the $(^{3}\text{He},^{3}\text{He}')$ and $(^{3}\text{He},\alpha)$ data for the dissimilar compound nuclei pairs, over the reported excitation energy range. The dissimilar compound nuclei pairs have neutron separation energies that differ by approximately 1.7 MeV ($S_{164}^{n} = 6454.4$ keV and $S_{162}^{n} = 8196.9$ keV). This implies that positive equivalent neutron energy for these surrogate measurements occurs well beyond the neutron separation energy for one of the compound nuclei employed in the ratio. The significant contribution to the nonstatistical error arising from the effects of potential target contamination for the dissimilar compound nuclei pairs is manifest because the shape of the correction varies significantly at excitation energies near $S_{n}$ as compared to those near $S_{n} + 1.7$ MeV.

The maximum statistical and nonstatistical uncertainty for the surrogate cross-section data for similar compound nuclei pairs over the excitation energy range of 8330 to 8690 keV was 22\%. The maximum total uncertainty for the surrogate cross-section data for dissimilar compound nuclei pairs over the excitation energy range of 8340 to 8760 keV was 81\% and 67\% for the $(^{3}\text{He},^{3}\text{He}')$ and $(^{3}\text{He},\alpha)$ data, respectively. It is important to note that these values represent a conservative estimate of the uncertainty in the surrogate cross-section data.

[24] M. Herman (private communication).